Math Mastery: An Approach to Algebra Skills Teaching and Assessment

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Summer 2019

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In 2013, we (The Roxbury Latin School, Boston, MA) replaced our 9th grade Algebra 2 course with a project-driven interdisciplinary STEAM course that we titled Math-Science Investigations (MSI). All of our 9th graders take the course in mixed non-leveled sections. I have historically taught two sections of the course, and a colleague teaches the other two sections. Feedback from our 10th grade math teachers after the first few years indicated that the students' basic algebra skills were not in all cases as strong as they were before introducing this new course. While this was not completely unexpected, my colleague and I responded to this feedback by evolving the course to include more intentional algebra instruction and review. Math subjects are interspersed throughout the projects in the first three marking periods, and the fourth marking period is now primarily math driven. Students take 4-6 math quizzes per marking period, and the mid-year and final exams primarily test math content. Marking period grades are calculated with roughly equivalent weight on project work and math quizzes.

For 30-40% of the students in each section, the math subjects are largely familiar review, and their performance on quizzes is excellent - indicative of a high level of mastery of the material. The middle 40-50% are certainly in need of the review that we offer, but are able to get up to speed quickly, and demonstrate a good-to-strong level of mastery. Some of the students in this group ask for opportunities to turn in test corrections in order to earn back missed points, but they will generally take their B to A- grade and move on. The remaining bottom of the class may not have seen the material before, often struggle to learn it in time for the quiz, and demonstrate a clear lack of mastery of the material. This group is often invited to take a re-take or to turn in quiz corrections, but since there has not historically been a formalized process for doing so, the class often moves along and the student neglects to take any action to improve their grade or their understanding. Students who ended up in the lowest group on quizzes often show little to no improvement on the same material on the mid-year and final exams.

The majority of our students end up taking some level of calculus before graduating, and nearly all students take four full years of high school math. The math skills that we are teaching and reinforcing in 9th grade are foundational to their subsequent study. If we allow some students to pass the class without some level of mastery of the basic skills, we are doing them and their future teachers a disservice. What I aim to realize is a system by which we can require students to reach a predetermined level of mastery of the material in order to pass the course. The challenges are to create a system in which the students are provided with sufficient opportunities to practice their skills, in which the pace of the material is appropriate for each student in a heterogeneous class, in which students are motivated to put in the effort needed to reach the mastery level, and in which the administrative and tutoring load of the instructor isn't increased to the point of non-viability. The research that I have explored looks at five stages of teaching and testing mastery (not always in this sequence): initial instruction, crafting assessments, giving assessments, remediation and accountability, and finally, grading.

Literature Review

Overview of Research

According to the well known educational psychologist Benjamin Bloom, "Most students (perhaps over 90 percent) can master what we have to teach them, and it is the task of instruction to find the means which will enable our students to master the subject under consideration. Our basic task is to determine what we mean by mastery of the subject and to search for the methods and materials which will enable the largest proportion of our students to attain such mastery" (1968). Bloom was an early advocate for the concept of mastery learning, which Dalton and Hannafin explain, "is essentially a method involving the teaching of ordered skills through a systematic cycle of teaching, testing, and remediating to criterion performance levels" (1988). While we often think of teaching broadly as encompassing this cycle in its totality, it is useful to break it down into its component parts in order to discover the most effective methods of teaching, testing, and remediating for mastery learning.

The initial teaching phase of this cycle can theoretically take any form, but in my context, content is presented in traditional teacher-led direct instruction. While for certain subjects, topics, and students this can be quite effective, alternative student-based instruction methods have been made possible through developments in computing. Studies have looked at the efficacy of computer-based instruction and practice as alternate modes of instruction and learning, with mixed results. Nevid and Gordon reference several studies that failed to produce clear results (2018), and in their research they aimed to improve on the inconsistencies and limitations of previous studies. Specifically, they were concerned with small sample sizes, failure to balance homework load between study groups, and weak incentives for engaging with online materials. Their study of an Integrated Learning System (ILS) "provides support for the learning benefits of ILS quizzing and concept building exercises when they are assigned as required homework to a mastery criterion level and constitute a substantial proportion of the course grade" (2018). Similarly to some of the studies that they criticised, they did not see strong benefits when use of the system was not highly incentivized by their course grade, or when mastery-level achievement was required. Mikula and Heckler studied the efficacy of "online instructional intervention," in a post-secondary physics course, but they targeted basic underlying skills instead of specific current course topics. They hypothesize that "problem solving requires accuracy and fluency in relatively simple, elementary skills..." and they argue "that students frequently do not have these simple yet essential skills, or they are far from fluent in their use" (2017). Through their students' use of their custom-designed software, their study provides "support for the idea that many students have difficulties with basic, essential skills in STEM courses—even postinstruction—and distributed, interleaved practice using computer-based, mastery graded instruction with immediate feedback can significantly—perhaps even dramatically—improve and maintain accuracy and fluency with these skills with only a few hours of practice during a semester course" (2017).

Following initial instruction and practice, mastery learning tests students' understanding with some form of assessment. Bres, Weisshaar, and Moore-Crawford propose a framework for developing assessments that focuses on developing goals for the course, establishing learning outcomes, ranking the outcomes, and finally selecting appropriate assessment tools based on the goals and ranked outcomes (2009). Tools include homework, exams, and quizzes, but can also include activities during class or lab time. They emphasize that designing assessments is an important opportunity to convene all relevant parties (for example, teachers of different sections of a course, department colleagues, department leadership) to clarify expectations and priorities. For assessments that are aimed to convey mastery, particular consideration needs to be given to course goals and desired outcomes. For example, a collection of “problem bank” or “back of the book” questions on a quiz may neglect to assess a student’s understanding of certain desired skills, and may ignore completely course goals related to synthesizing information.

Based on the results of assessments, remediation is prescribed to students that have not yet met the stated learning objectives. One common criticism of mastery learning is that the one-on-one remediation between teacher and student is unrealistically time-intensive. Bloom writes, “Were it not so costly in human resources, we believe that the provision of a good tutor for each student might be one ideal strategy. In any case, the tutor-student relationship is a useful model to consider when one attempts to work out the details of a less costly strategy” (1968). Dalton and Hanafin, no doubt referencing Bloom’s work in part, claim that “mastery learning advocates have benchmarked individualized one-to-one tutoring as the ideal for group-administered mastery instruction. Mastery methods ideally should approach the effectiveness and efficiency of one-to-one tutoring” (1988). They propose that computer-assisted technologies that can be automatically adapted to students’ real-time performance might be able to approach the one-to-one ideal, without adding additional work for the instructor. They also determined that “Variations in delivery system, whether from computer system to traditional system, or traditional system to computer system, provided the greatest impact on lesson achievement. Students performed best when the delivery system employed for the remedial strategy was different from the system for initial instruction” (1988). Diegelman-Parente agrees that computer-assisted adaptive instruction can take some of the pressure off the instructor when providing remediation. She suggests that peer tutoring can be a high-quality method of remediation for struggling students as well, and that it also provides reinforcement and practice to the tutor (2011). In addition, she advises that students that achieve mastery quickly shouldn't be ignored while offering remediation to other students, rather, enrichment opportunities should be provided to them, such as opportunities to apply knowledge to "real-world" problems, peer tutoring, or bonus problems.

Time is often the most valuable currency in schools, and the issue of time becomes a substantial factor at this point in the mastery learning process. Bloom's initial research on time and learning found that students may differ substantially in how much time it takes to learn given material - "by a ratio of about 5:1 under a variety of learning conditions" (1974). Bloom argues that mastery learning training can shrink the ratio, but teachers are always likely to encounter students that require different amounts of time to achieve the same level of mastery. Where that time is spent (in the classroom, at school, at home, etc...) is an important variable to explore when imagining a mastery learning program. Diegelman-Parente claims that "Whereas some mastery learning strategies do incorporate methods that alter the pace of instruction, this is not a required tenet of Bloom's mastery learning strategy. Indeed, much of Bloom's comparative data on mastery learning and control classes has students experiencing the same schedule of instruction with corrective work of the mastery students (when necessary) done outside of the classroom" (2011). While it may not be a "required tenet," Diegelman-Parente in the same article cites Block (a student of Bloom's), saying, "Mastery learning, as refined by Block (1971), is an instructional method presuming that students learn best if they fully understand, or master, one concept before going on to the next." It would seem as though the inability to adapt the schedule of instruction to particular students' needs is a compromise inherent in our current educational structure, rather than an easily dismissed non-factor. Realistically, most mastery learning applications *do* rely heavily on student time outside of the classroom for remediation. Moreover, additional student time is required for students to actually take retests. Finding or allocating this time on behalf of their students is a challenge for many educators attempting to implement mastery learning. Time between retests should be defined, (Diegelman-Parente's course requires a mandatory 48-hour waiting period before retests, and retests must occur within a week of any previous test) but it will depend heavily on the content and format of the course. Rather than dwelling on the time between retests, attention should be paid to the prescribed remediation steps between tests, and the ways in which teachers can promote student accountability and motivation to take advantage of remediation opportunities.

Generally, students fail to independently take advantage of provided enrichment and remediation resources if their use does not contribute meaningfully to a grade (Mikula and Heckler 2017, Nevid and Gordon 2018). Mikula and Heckler's study compared the difference between awarding course credit versus extra credit, and found substantially higher use of their online tool when course credit was awarded. Nevid and Gordon compared the use of an Integrated Learning System (ILS) across two groups, one in which mastery completion of the ILS quizzes was worth 42% of their overall grade, and the other in which use of the ILS was optional. They found a striking (though not surprising) difference; "the ILS-required class maintained a consistently high rate of utilization (based on submission of chapter quizzes) across the semester, whereas the ILS-optional class showed a consistent downward pattern that approached a near zero rate by the end of the semester" (2018). While it appears that grades are a generally useful tool for student accountability, many advocates of mastery learning strongly prefer standards-based or competency-based grades as an alternative to traditional grades.

Lehman, De Jong, and Baron compared the relationship of standards-based grades and traditional-based grades with standardized test scores in a population of middle school mathematics students, and found that "all standards-based grades in this study correlated more highly to SMI [standardized math test scores] than corresponding traditional grades" (2018). They blame the subjectivity of traditional grades, and the attempts to incorporate "both content and effort" into a single metric. "Austin and McCann argued that when educators do not agree on the primary purpose of grades, they often try to address all of these purposes with a single reporting device, usually a report card, and end up achieving none very well (as cited in Guskey, 2015)." They cite Hanover Research (2011) in describing several common practices that "prevent grades from being accurate measures of students' performance," including "point systems and averages, zeros as punishment, grading homework and formative assessments, grading on a curve, allowing extra credit, grading for behavioral issues, [and] incorporating teacher expectations and judgements into grades." They recommend that all leaders of mathematics education consider moving to standards-based grades.

Conclusion

There is substantial evidence that mastery-based learning, particularly in STEM subject areas, is a more equitable, complete, and long-lasting approach to learning than more traditional approaches that don't allow for supported remediation and confirmation of eventual mastery. It is not implemented as widely as perhaps it should be because of the challenges of providing individual support to large classes of diverse learners, and the inability or unwillingness to change the pace of introduction of new material. Classrooms are often populated by students with broadly diverse educational backgrounds, and mastery of a cumulative subject is particularly difficult if some students don't yet have a sufficient level of fluency with prerequisite skills. Generating, grading, and providing feedback on multiple iterations of similar assessments is likely to be time-intensive, and if teachers imagine offering traditional assessments in this way, it is understandably daunting. Meanwhile, online teaching and learning tools, particularly in STEM fields, have shown promise. If students are required to utilize them as part of the course and also required to achieve a level of mastery as judged by the tool, evidence shows that learning goals on subsequent assessments are more likely to be met. Adaptive learning tools are able to quickly determine whether a student needs continued practice with a skill or whether they have mastered it and are prepared to move on. Utilizing adaptive tools to provide remediation or even assessment of certain skills may take some of the time pressure off of teachers, and provide fruitful modes of self-directed and independent remediation for students. Use of these tools may help to dismantle some of the barriers to mastery learning, and allow for greater adoption of mastery learning pedagogies. Standards-based or competency-based grading dovetails nicely with mastery learning in order to accurately communicate students' understanding and academic achievement, by excluding traditional obfuscating factors. While many schools are likely to compromise to find a way to translate standards-based grades (for internal communication) into traditional grades (for external communication), this will impinge on the utility of the grading scheme. Ideally, schools would move completely to standards-based grades.

Annotated Bibliography

Bres, M., Weisshaar, A., & Moore-Crawford, C. (2009). Streamlining assessment. *Journal of College Science Teaching, 38*(5), 43-47. Retrieved from http://ezproxy.msu.edu.proxy2.cl.msu.edu/login?url=https://search-proquest-com.proxy2.cl.msu.edu/docview/61862364?accountid=12598

The authors of this article come from a higher education biology background, but propose a framework for assessments that are transferable to any discipline. They suggest a system of developing goals, establishing learning outcomes, ranking their relative importance, and finally selecting appropriate assessment tools. As I try to imagine what exactly it is that my students need to master in my course, it will be important to carefully reevaluate the goals and learning outcomes of the course. Carefully defining these goals will allow the quiz/assessment generation process to be much more streamlined, efficient, and ideally, effective. The framework that they suggest may be useful in this way.

Dalton, D. W., & Hannafin, M. J. (1988). The effects of computer-assisted and traditional mastery methods on computation accuracy and attitudes. *Journal of Educational Research, 82*(1), 27-33. Retrieved from http://ezproxy.msu.edu.proxy2.cl.msu.edu/login?url=https://search-proquest-com.proxy2.cl.msu.edu/docview/63112677?accountid=12598

It is interesting how a discussion of computer-assisted methods in mathematics that was written in 1988 can still read so relevantly in 2019. The authors designed an experiment to test the effectiveness of computer- and teacher-based methods of mastery teaching. Their study was partly motivated by the common critiques of mastery teaching methods, which revolve around it being too difficult to implement realistically, or inequitably beneficial in large groups. The goal was to teach how to compute the area of a circle to 8th graders. While there was no clear evidence of superiority between computer- and teacher-based instruction, the mastery-based instruction resulted in significantly improved achievement, and most notably so when the initial instruction and remediation instruction were varied between computer and teacher. (Computer then teacher, or teacher then computer)

Diegelman-Parente, A. (2011). The use of mastery learning with competency-based grading in an organic chemistry course. *Journal of College Science Teaching, 40*(5), 50-58. Retrieved from http://ezproxy.msu.edu.proxy2.cl.msu.edu/login?url=https://search-proquest-com.proxy2.cl.msu.edu/docview/1018482460?accountid=12598

This article details the author's attempt to create a grading scheme that complements her mastery-learning based pedagogy in her university-level Organic Chemistry course. They key to her mastery learning philosophy is building appropriate assessment, feedback, correction, enrichment, assessment loops that are clear to the students and minimally invasive to the instructor. She allows students to earn a low passing grade (C) by meeting minimum mastery standards on all assessments. Students can then earn higher grades if they choose to take on enrichment or extension activities, such as demonstrating higher levels of understanding, solving bonus problems, attending class, applying concepts to "real-world" problems, and tutoring peers. Anyone that fails to meet mastery level as determined by the professor receives an F.

Lehman, E., De Jong, D., & Baron, M. (2018). Investigating the relationship of standards-based grades vs. traditional-based grades to results of the scholastic math inventory at the middle school level. *Education Leadership Review of Doctoral Research, 6*, 1-16. Retrieved from http://ezproxy.msu.edu.proxy2.cl.msu.edu/login?url=https://search-proquest-com.proxy2.cl.msu.edu/docview/2228656779?accountid=12598

It seems as though mastery learning requires a parallel reevaluation of grading systems, in order to accurately reflect the meaning of the grade. This study describes many of the authors' perceived problems with traditional grading that contribute to inaccuracy, including a lack of standards, lack of objectivity, extra credit, homework grades, and grading as punishment. They show that in the school that they studied, there is little correlation between traditional math grades and standardized test scores (Scholastic Math Inventory, or SMI), and significant positive correlations between alternative standards-based grading and SMI. They recommend that all leaders of mathematics education consider moving to standards-based grades.

Mikula, B. D., & Heckler, A. F. (2017). Framework and implementation for improving physics essential skills via computer-based practice: Vector math. *Physical Review Physics Education Research, 13*(1), 010122-1. doi:http://dx.doi.org.proxy2.cl.msu.edu/10.1103/PhysRevPhysEducRes.13.010122

This study could be oversimplified to "practice makes perfect," but the study and resulting data is actually quite interesting. The researchers designed an online problem solving system to reinforce basic vector math skills that were essential to success in a post-secondary introductory calculus-based physics course. They aimed to study the ways in which the essential skills (ES) practice affected students' accuracy and fluency with those skills. Since the mathematics taught in this class is largely simple skills taught in preparation for future courses, this study is particularly applicable. The online system was designed to require mastery of a topic, in this case defined as three correct answers to a particular type of problem without an incorrect answer. If a student answered incorrectly, the count returned to zero. The result was that students who used the online system showed substantially significant improvements in both accuracy and fluency with the skills, even though the total training time (median 105 minutes throughout a semester) was relatively small. Interesting relative to the effect of grading on motivation, students for whom the ES practice counted as course credit performed more training assignments and improved more than those for whom it counted as extra credit.

Nevid, J. S., & Gordon, A. J. (2018). Integrated learning systems: Is there a learning benefit? *Teaching of Psychology, 45*(4), 340-345. doi:http://dx.doi.org.proxy2.cl.msu.edu/10.1177/0098628318796920

Similarly to Mikula & Heckler's study, this study looks to evaluate the effectiveness of Integrated Learning Systems (ILS), in this case an online quizzing system that is packaged with the textbook for an introductory psychology course. They quote several other studies that showed little to no positive correlation between the use of an ILS and course success, but they point out that there was little incentive for students to use the system, and little to no requirement of mastery of the material in the ILS. They designed an experiment in which students were given substantial course credit for completing the online quizzes, and required a mastery level of understanding (in this case, a perfect score) in order to receive credit. The control group had access to the online resources, but received no credit for utilizing it. The results showed that requiring the online quizzes, and requiring completion at a mastery level, resulted in significantly higher exam performance than the control group. Quizzes were untimed and students were allowed unlimited attempts.

Resources

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